

**TEMA 1.**

**Resolución de ejercicios de números enteros, fraccionarios, exponenciales y radicales.**

1. Realiza las operaciones entre números enteros indicadas:

a)  $(3-5)-(2+5)=-2-7=-9$

b)  $4+3+(3+1)=7+4=11$

c)  $2+4-6-(2+6+4)=2+4-6-12=-12$

d)  $4-3-(-2-3+1)-2=1-(-4)=1+4=5$

e)  $7-[3+(4-1)-(2-1)]=7-(3+3-1)=7-5=2$

f)  $[(3+2)-(5-3)]-[(4-2)-3]=(5-2)-(2-3)=3-(-1)=3+4=7$

2. Realiza las siguientes operaciones entre números enteros:

a)  $4 \cdot (-3) = -12$

b)  $2 \cdot 3 \cdot (-3) = -18$

c)  $-1 \cdot (-2) + (-2) \cdot (-3) = 2 + 6 = 8$

d)  $4 \cdot [(3+2)-5] = 4 \cdot (5-5) = 4 \cdot 0 = 0$

e)  $2 \cdot [(4-2)-3 \cdot (-2)] = 2 \cdot (2+6) = 2 \cdot 8 = 16$

f)  $(3-2) \cdot (1-6) - 5 \cdot (2-1) = 1 \cdot (-5) - 5 \cdot 1 = -5 - 5 = -10$

3. Realiza las siguientes operaciones con fracciones y simplifica cuando sea posible:

a)  $\frac{1}{2} + \frac{2}{3} = \frac{3+4}{5} = \frac{7}{5}$

b)  $\frac{1}{3} - \frac{4}{9} + 1 = \frac{9-4+1}{9} = \frac{6}{9} = \frac{2}{3}$

c)  $\frac{4}{5} - \frac{7}{12} - \frac{3}{10} = \frac{4 \cdot 12 - 7 \cdot 5 - 3 \cdot 6}{60} = \frac{48 - 35 - 18}{60} = -\frac{5}{60} = -\frac{1}{12}$

d)  $\frac{3}{4} - \frac{5}{2} = \frac{3-5 \cdot 2}{4} = \frac{3-10}{4} = -\frac{7}{4}$

$$e) \frac{4}{25} + \frac{1}{30} - \frac{7}{15} = \frac{4 \cdot 6 - 1 \cdot 5 - 7 \cdot 10}{150} = \frac{24 - 5 - 70}{150} = -\frac{51}{150}$$

$$f) \frac{7}{60} - \frac{3}{40} - 2 - \frac{1}{50} = \frac{7 \cdot 6 - 3 \cdot 15 - 2 \cdot 60 - 1 \cdot 12}{600} = \frac{42 - 45 - 120 - 12}{600} = -\frac{135}{600} = -\frac{9}{40}$$

4. Calcula las inversas de las siguientes fracciones:

$$a) \frac{1}{2}. \text{ La inversa de } \frac{1}{2} \text{ es } \left(\frac{1}{2}\right)^{-1} = \frac{1}{\frac{1}{2}} = 2$$

$$b) \frac{3}{2}. \text{ La inversa de } \frac{3}{2} \text{ es } \left(\frac{3}{2}\right)^{-1} = \frac{1}{\frac{3}{2}} = \frac{2}{3}$$

$$c) \frac{11}{5}. \text{ La inversa de } \frac{11}{5} \text{ es } \left(\frac{11}{5}\right)^{-1} = \frac{1}{\frac{11}{5}} = \frac{5}{11}$$

$$d) \frac{15}{44}. \text{ La inversa de } \frac{15}{44} \text{ es } \left(\frac{15}{44}\right)^{-1} = \frac{1}{\frac{15}{44}} = \frac{44}{15}$$

$$e) \frac{2}{123}. \text{ La inversa de } \frac{2}{123} \text{ es } \left(\frac{2}{123}\right)^{-1} = \frac{1}{\frac{2}{123}} = \frac{123}{2}$$

$$f) \frac{123}{2}. \text{ La inversa de } \frac{123}{2} \text{ es } \left(\frac{123}{2}\right)^{-1} = \frac{1}{\frac{123}{2}} = \frac{2}{123}$$

5. Desarrolla las siguientes potencias:

$$a) 2^2 = 2 \cdot 2 = 4$$

$$b) 3^2 = 3 \cdot 3 = 9$$

$$c) 5^3 = 5 \cdot 5 \cdot 5 = 125$$

$$d) 11^2 = 11 \cdot 11 = 121$$

$$e) (-2)^2 = (-2) \cdot (-2) = 4$$

$$f) (-2)^3 = (-2) \cdot (-2) \cdot (-2) = -8$$

$$g) (-3)^4 = (-3) \cdot (-3) \cdot (-3) \cdot (-3) = 81$$

6. Realiza las siguientes operaciones de potencias:

$$a) 2^2 \cdot 2^3 = 2^{2+3} = 2^5$$

$$b) 3^3 \cdot 3^4 = 3^{3+4} = 3^7$$

$$c) 11^4 \cdot 11 = 11^{4+1} = 11^5$$

$$d) 3^2 \cdot 3 = 3^{2+1} = 3^3$$

$$e) 5^2 \cdot 5^{-1} = 5^{2+(-1)} = 5^1 = 5$$

$$f) 5^2 \cdot 5^3 \cdot 5^{-2} = 5^{2+3+(-2)} = 5^3$$

$$g) 2^3 \cdot 2^{\frac{1}{2}} = 2^{3+\frac{1}{2}} = 2^{\frac{7}{2}} = \sqrt{2^7}$$

$$h) 2^{\frac{1}{3}} \cdot 2^{\frac{3}{4}} = 2^{\frac{1+3}{3+4}} = 2^{\frac{4+9}{12}} = 2^{\frac{13}{12}} = \sqrt[12]{2^{13}}$$

$$i) 2^{\frac{1}{2}} \cdot 2^{\frac{2}{3}} \cdot 2^{-2} = 2^{\frac{1+2}{2+3}-2} = 2^{\frac{-3+4-12}{6}} = 2^{-\frac{11}{6}} = \frac{1}{2^{\frac{11}{6}}} = \frac{1}{\sqrt[6]{2^{11}}}$$

7. Simplifica. Expresa el resultado en forma de raíz cuando el exponente sea fraccionario

$$a) \frac{2^5 \cdot 2^4 \cdot 2^{-2}}{2^2 \cdot 2^3} = \frac{2^{5+4-2}}{2^{2+3}} = \frac{2^7}{2^5} = 2^2$$

$$b) \frac{2^2 \cdot 2^3 \cdot 2^{-2}}{2 \cdot 2^3 \cdot 2^{-6}} = \frac{2^{2+3-2}}{2^{1+3-6}} = \frac{2^{-4}}{2^{-2}} = 2^{-4-(-2)} = 2^{-2} = \frac{1}{2}$$

$$c) \frac{2^2 \cdot 2^{\frac{3}{2}} \cdot 2}{2^{\frac{1}{2}} \cdot 2^3 \cdot 2^{-1}} = \frac{2^{2+\frac{3}{2}+1}}{2^{\frac{1}{2}+3-1}} = \frac{2^{\frac{7}{2}}}{2^{\frac{5}{2}}} = 2^{\frac{7-5}{2}} = 2^1 = 2$$

$$d) \frac{\sqrt{2} \cdot \sqrt[3]{2}}{2^3 \cdot \sqrt[5]{2}} = \frac{2^{\frac{1}{2}} \cdot 2^{\frac{1}{3}}}{2^3 \cdot 2^{\frac{1}{5}}} = \frac{2^{\frac{5}{6}}}{2^{\frac{16}{5}}} = 2^{\frac{5-71}{30}} = 2^{-\frac{71}{30}} = \frac{1}{2^{\frac{71}{30}}} = \frac{1}{\sqrt[30]{2^{71}}}$$

$$e) \frac{\sqrt{2^3} \cdot \sqrt[3]{2^4}}{2^3 \cdot \sqrt[5]{2^6}} = \frac{(2^3)^{\frac{1}{2}} \cdot (2^4)^{\frac{1}{3}}}{2^3 \cdot (2^6)^{\frac{1}{5}}} = \frac{2^{\frac{3}{2}} \cdot 2^{\frac{4}{3}}}{2^3 \cdot 2^{\frac{6}{5}}} = \frac{2^{\frac{17}{6}}}{2^{\frac{21}{5}}} = 2^{\frac{17-41}{30}} = 2^{-\frac{41}{30}} = \frac{1}{2^{\frac{41}{30}}} = \frac{1}{\sqrt[30]{2^{41}}}$$

$$f) \frac{\sqrt{2^{-2}} \cdot \sqrt[4]{2^4} \cdot \sqrt[7]{2^{14}}}{2^{-1} \cdot \sqrt[6]{2^3}} = \frac{(2^{-2})^{\frac{1}{2}} \cdot (2^4)^{\frac{1}{4}} \cdot (2^{14})^{\frac{1}{7}}}{2^{-1} \cdot (2^3)^{\frac{1}{6}}} = \frac{2^{-1} \cdot 2 \cdot 2^2}{2^{-1} \cdot 2^{\frac{1}{2}}} = \frac{2^2}{2^{-\frac{1}{2}}} = 2^{\frac{5}{2}} = \sqrt{2^5}$$

8. Opera con frazioni:

$$a) \frac{1}{2} \cdot \frac{3}{5} = \frac{3}{10}$$

$$b) \frac{3}{5} \cdot \frac{9}{4} = \frac{27}{20}$$

$$c) \frac{1}{3} \cdot \frac{7}{2} \cdot \frac{2}{5} = \frac{7}{15}$$

$$d) \frac{1}{3} \cdot \frac{(-7)}{2} \cdot \frac{(-2)}{5} = \frac{7}{15}$$

$$e) \frac{1}{2} : \frac{3}{5} = \frac{5}{6}$$

$$f) \frac{3}{5} : \frac{9}{4} = \frac{3 \cdot 4}{5 \cdot 9} = \frac{4}{15}$$

$$g) \frac{1}{3} : \frac{(-7)}{2} = -\frac{2}{21}$$

$$h) \frac{(-7)}{2} : \frac{(-2)}{5} = \frac{35}{4}$$

$$i) \frac{\frac{1}{2}}{\frac{2}{3}} = \frac{1}{2} : \frac{2}{3} = \frac{3}{4}$$

$$j) \frac{\frac{1}{2}}{\frac{5}{6}} = \frac{1}{2} : \frac{5}{6} = \frac{3}{5}$$

$$k) \frac{2}{\frac{1}{3}} = 2 : \frac{1}{3} = 6$$

$$l) \frac{\frac{2}{3}}{\frac{4}{3}} = \frac{2}{3} : 4 = \frac{2}{12} = \frac{1}{6}$$

$$m) \frac{\frac{1}{2} \cdot \frac{3}{5}}{\frac{2}{3} \cdot \frac{1}{7}} = \frac{\frac{3}{10}}{\frac{2}{21}} = \frac{3}{10} \cdot \frac{21}{2} = \frac{63}{20}$$

$$n) \frac{1}{2} - \left( \frac{3}{5} - 1 \right) = \frac{1}{2} - \left( \frac{-2}{5} \right) = \frac{1}{2} + \frac{2}{5} = \frac{9}{10}$$

$$\tilde{n}) \frac{1}{2} - \left\{ \left( \frac{3}{5} - 1 \right) + \frac{3}{2} \right\} = \frac{1}{2} - \left( -\frac{2}{5} + \frac{3}{2} \right) = \frac{1}{2} - \frac{11}{10} = -\frac{6}{10} = -\frac{3}{5}$$

$$o) \frac{1}{2} \cdot \left( \frac{2}{7} - \frac{1}{14} - 2 \right) = \frac{1}{2} \cdot \left( \frac{2 \cdot 2 - 1 - 2 \cdot 14}{14} \right) = \frac{1}{2} \cdot \left( -\frac{25}{14} \right) = \frac{7 + 25}{14} = \frac{32}{14} = \frac{16}{7}$$

$$p) \frac{1}{2} : \left( 1 - \frac{1}{5} - \frac{2}{25} \right) - 1 = \frac{1}{2} : \left( \frac{25 - 5 - 2}{25} \right) - 1 = \frac{1}{2} : \left( \frac{18}{25} \right) - 1 = \frac{25}{32} - 1 = -\frac{7}{32}$$

$$q) 1 + \frac{1}{2} = \frac{2+1}{2} = \frac{3}{2}$$

$$r) 1 + \frac{1}{1 + \frac{1}{2}} = 1 + \frac{1}{\frac{3}{2}} = 1 + \frac{2}{3} = \frac{5}{3}$$

$$s) \frac{1}{2} : \left( 1 + \frac{1}{1 + \frac{1}{2}} \right) = \frac{1}{2} : \frac{5}{3} = \frac{3}{10}$$

9. Calcula las siguientes potencias, expresando el resultado de la forma  $a^b$

$$a) \left( \frac{1}{3} \right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$$

$$b) \left( -\frac{1}{3} \right)^2 = \frac{1^2}{3^2} = \frac{1}{9}$$

$$c) \left( -\frac{1}{3} \right)^3 = -\frac{1^3}{3^3} = -\frac{1}{27}$$

$$d) \left( \frac{1}{3} \right)^{-2} = \frac{1}{\left( \frac{1}{3} \right)^2} = \frac{1}{\frac{1}{9}} = 9$$

$$e) \left( -\frac{1}{3} \right)^4 = \frac{1}{3^4} = \frac{1}{81}$$

$$f) \left(\frac{1}{2}\right)^{-3} = \frac{1}{\left(\frac{1}{2}\right)^3} = \frac{1}{\frac{1}{8}} = 8$$

$$g) \left(-\frac{1}{3}\right)^0 = 1$$

$$h) \left(\frac{1}{5}\right)^{-2} = \frac{1}{\left(\frac{1}{5}\right)^2} = \frac{1}{\frac{1}{25}} = 25$$

10. Reduce cada una de las siguientes expresiones a la forma  $\sqrt[n]{a^n}$

$$a) 2^{\frac{1}{2}} \cdot \left(\left(3^{\frac{1}{2}}\right)^3\right)^{\frac{1}{2}} = 2^{\frac{1}{2}} \cdot 3^{\frac{3}{4}} = \sqrt{2} \cdot \sqrt[4]{3^3}$$

$$b) \left(\left(3^{\frac{1}{2}}\right)^3\right)^{\frac{1}{2}} \cdot \left(\left(3^2\right)^3\right)^{\frac{1}{2}} = 3^{\frac{3}{4}} \cdot 3^3 = 3^{\frac{15}{4}} = \sqrt[4]{3^{15}}$$

$$c) \left(\left(2^{\frac{1}{3}}\right)^{-3}\right)^{\frac{1}{2}} = 2^{-\frac{1}{2}} = \frac{1}{2^{\frac{1}{2}}} = \frac{1}{\sqrt{2}}$$

$$d) \sqrt{\sqrt{2}} = \left(2^{\frac{1}{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{4}} = \sqrt[4]{2}$$

$$e) \sqrt{\sqrt{\sqrt{2}}} = \left(\left(2^{\frac{1}{2}}\right)^{\frac{1}{2}}\right)^{\frac{1}{2}} = 2^{\frac{1}{8}} = \sqrt[8]{2}$$

$$f) \left(\sqrt[5]{\sqrt{2}}\right)^2 = \left(\left(2^{\frac{1}{2}}\right)^{\frac{1}{5}}\right)^2 = 2^{\frac{2}{15}} = \sqrt[15]{2^2}$$

$$g) \left(\sqrt[3]{\sqrt[3]{\left(\sqrt[6]{2^3}\right)^{\frac{1}{2}}}}\right)^2 = \left(\left(\left(2^3\right)^{\frac{1}{6}}\right)^{\frac{1}{3}}\right)^{\frac{1}{3}}\right)^2 = 2^{\frac{1}{9}} = \sqrt[9]{2}$$